Math 128a – Week 12 Worksheet
GSI: Izak, (4/14/21)

5.9 Problems

Problem 1. *Use the Runge-Kutta method for systems to approximate the solutions of the following system of first-order differential equations and compare the results to the actual solutions.*

$u_1' = -4u_1 - 2u_2 + \cos(t) + 4\sin(t)$, $u_2' = 3u_1 + u_2 - 3\sin(t)$, $u_1(0) = 0$, $u_2(0) = -1$, $0 \leq t \leq 2$, $h = .1$. *Actual solutions* $u_1(t) = 2e^{-t} - 2e^{-2t} + \sin(t)$ *and* $u_2(t) = -3e^{-t} + 2e^{-2t}$

5.10 Problems

Problem 2. *Show that if the initial value problem* $y' = f(t, y)$, $a \leq t \leq b$, $y(a) = \alpha$ *is approximated by a one-step difference method:* $w_0 = \alpha$, $w_{i+1} = w_i + h\varphi(t_i, w_i, h)$. *And there exists* $h_0 > 0$ *and* $\varphi(t, w, h)$ *continuous and Lipschitz in the* w *variable with Lipschitz constant* L *on*

$$D = \{(t, w, h) | a \leq t \leq b, -\infty < w < \infty, 0 \leq h \leq h_0\}$$

then there exists a constant $K > 0$ *such that*

$$|u_i - v_i| \leq K|u_0 - v_0|$$

for each $1 \leq i \leq N$ *whenever* $\{u_i\}_{i=1}^N$ *and* $\{v_i\}_{i=1}^N$ *satisfy the difference equation* $w_{i+1} = w_i + h\varphi(t_i, w_i, h)$

Problem 3. *Show that Runge-Kutta method of order four is consistent using the results of exercise 32 in section 5.4.*

Problem 4. *Given the multistep method*

$$w_{i+1} = -\frac{3}{2}w_i + 3w_{i-1} - \frac{1}{2}w_{i-2} + 3hf(t_i, w_i)$$

for $i = 2, \ldots, N-1$, *with starting values* w_0, w_1, w_2. *(a) Find the local truncation error. (b) comment on consistency, stability, and convergence.*